Turbulent Flow in Wetlands with Submerged and Floating Vegetation

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Abstract—Turbulent Flow in wetlands with submerged and floating aquatic plants was studied by applying Boit’s poroelastic theory to turbulent flow. The velocity distribution formulas were described by solving the modified Bessel equations. The effects of plant porosity and turbulent coefficient on the velocity distribution, shear stress distribution and energy relations were analytically derived. Several illustrative cases were given to show the details of the applications. The results of the study can help water resources, environmental and ecological engineers design and manage surface water flows in natural and constructed wetlands.

Index Terms—Flow characters, Vegetation flow, Porous medium flow, Velocity distribution, Shear stress distribution.

I. INTRODUCTION

Natural wetlands with submerged and floating aquatic plants act as biofilter, removing sediments and pollutants. Studies on Flows in such wetlands are very useful.

The early studies of the subject focused on the vegetation resistance coefficients, such as the effects of vegetation on the Manning’s roughness coefficient, Darcy-Weisbach friction factor and drag coefficient[1-2]. Evaluation of various vegetation resistance formulas for flood management was documented by Galema[3]. Analytic models on the turbulent velocity distribution over a vegetated ground were proposed by Kubrak et al.[4]; Vegetation related sediment transport was studied by Wu et al.[5].

Beavers and Joseph[6] tried to solve the problem of the two-dimensional Poiseuille flow over a fluid-saturated and permeable porous medium. Song and Huang[7] further proposed complete governing equations and boundary conditions for porous medium flow. White B.L.and Nepf H.M. [8], Ping-Cheng Hsieh et al. [9] studied the Surface Water Flow over Vegetated Ground by applying Boit’s poroelastic theory[10].

Based on the foregoing comments, we found that most studies were executed from an empirical formula of the resistance, and the mechanism of flow through vegetation was not described in detail. Therefore, the application of poroelastic theory to the study of the turbulent flow with floating plants and submerged plants is of important in revealing.

In this paper, we consider a wetland flow with both submerged and floating plants. We apply Boit’s poroelastic theory of laminar to turbulent flow. We will derive not only the velocity distribution, but also the shear stress distribution and energy relations.

II. GOVERNING EQUATIONS

A. Governing equations for the turbulent poroelastic medium flow

Figure 1 illustrates the fully developed turbulent poroelastic medium flow. h is the thicknesses of the Poroelastic Medium, θ is the bed slope. The coordinates of the impervious bed and free surface are y=0 and y=h, respectively.

Referring to the work of Song and Huang [7], the unsteady poroelastic medium flow can be described by Biot’s theory of poroelasticity as follows:

\[(1-n)\rho \frac{\partial^2 \mathbf{d}}{\partial t^2} = \nabla \cdot \mathbf{\sigma}_s + \mu \frac{n^2}{k} \left( \frac{\partial \mathbf{D}}{\partial t} - \frac{\partial \mathbf{d}}{\partial t} \right); \quad (1)\]

\[n \rho \frac{\partial^2 \mathbf{D}}{\partial t^2} = \nabla \cdot \mathbf{\sigma}_f - \mu \frac{n^2}{k} \left( \frac{\partial \mathbf{D}}{\partial t} - \frac{\partial \mathbf{d}}{\partial t} \right); \quad (2)\]

\[\mathbf{\tau}_s = \mathbf{\tau}_e - (1-n)\rho \mathbf{I}; \quad (3)\]

\[\mathbf{\tau}_e = 2\mu \mathbf{e} + \lambda (\nabla \cdot \mathbf{d}) \mathbf{I}; \quad (4)\]

\[\mathbf{e} = \frac{1}{2} (\nabla \mathbf{d} + (\nabla \mathbf{d})^T) ; \quad (5)\]

\[\mathbf{\sigma}_f = n\mu \mathbf{I} + n\mu (\frac{\partial \mathbf{D}}{\partial t} + (\nabla \frac{\partial \mathbf{D}}{\partial t})^T) ; \quad (6)\]

\[\mathbf{u} = \frac{\partial \mathbf{D}}{\partial t} ; \quad (7)\]

where \(\mathbf{\sigma}_s\) is the total stress tensors of the solid; \(\mathbf{\sigma}_f\) is the total stress tensors of the fluid; \(\mathbf{\tau}_e\) is the effective stress tensor of the solid; \(\mathbf{e}\) is the strain tensor of the solid; \(\mathbf{d}\) and \(\mathbf{D}\) are the displacements of solid skeleton and fluid; \(\mathbf{u}\)
is the velocity vector of fluid; \( \mu \) is the viscosity of fluid; \( p \) is the fluid pressure; \( \rho \) is the fluid density; \( n \) and \( k \) are the porosity and special permeability of the porous medium; \( G \) and \( \lambda \) are the Lame constants of elasticity; \( t \) is the time; \( \Gamma \) is the transpose of the matrix; and \( I \) is the identity matrix.

Assuming that the flow is a steady uniform one and the solid grains will not be moved by the stream flow, we can omit the equations (1) and (4), and obtain the continuity equation from the equation (2):

$$\frac{\partial u}{\partial x} = 0,$$  \hspace{1cm} (8)

and the momentum equation:

$$n\mu \frac{d^2u}{dy^2} - \frac{\mu n^2}{k} u - \frac{d(n pu v)}{dy} - n \frac{\partial p}{\partial x} + n \rho g \sin \theta = 0;$$ \hspace{1cm} (9)

$$\frac{\partial p}{\partial y} + \rho g \cos \theta = 0,$$ \hspace{1cm} (10)

where \( u \) is the tangential component of the velocity vector \( \mathbf{u} \). If assume that \( p \) is distributed as hydrostatic pressure, the momentum equation (9) is:

$$n\mu \frac{d^2u}{dy^2} - \frac{dn pu v}{dy} - \frac{\mu n^2}{k} u + n \rho g \sin \theta = 0.$$ \hspace{1cm} (11)

Applying one of the turbulence models for the relation of Reynolds stresses in turbulence theory\textsuperscript{[11]}:

$$-n \rho pu v = n \beta \sin \theta \frac{y}{h} \frac{du}{dy},$$ \hspace{1cm} (12)

where \( \beta \) is the coefficient of turbulent. Substituting (12) into (11), we have

$$n\mu \frac{d^2u}{dy^2} + \frac{dn pu v}{dy} - \frac{\mu n^2}{k} u + n \rho g \sin \theta = 0.$$ \hspace{1cm} (13)

B. Governing equations for the flow with floating and submerged plants

Figure 2 illustrates steady fully developed flow with floating and submerged plants. \( h_2 \) and \( h_4 \) are the thicknesses of the lower plant layer 2 and upper plant layer 4, respectively; \( h_1 \) and \( h_3 \) are the thicknesses of the water layer 1 and 3. The velocity reaches to the maximum value at the interface between the water layer 1 and water layer 3. The top surface is parallel to the impervious bed.

For the lower plant layer 2 and water layer 1, the coordinates of the impervious bed and the interface between the lower plant layer 2 and water layer 1 are \( y=0 \) and \( y=h_2 \). According to the equation (13), the control equations in the lower plant layer 2 and water layer 1 can be written as:

$$n\mu \frac{d^2u_2}{dy^2} + \frac{dn u_2}{dy} \frac{n \beta \sin \theta}{h_2} \frac{y}{dy} 
- \frac{\mu n^2}{k_2} u_2 + n \rho g \sin \theta = 0;$$ \hspace{1cm} (14)

$$\mu \frac{d^2u_1}{dy^2} + \frac{d(n \beta \sin \theta)}{h_2} \frac{y}{dy} \frac{du_1}{dy} + \rho g \sin \theta = 0.$$ \hspace{1cm} (15)

The dimensionless forms of the equations (14) and (15) are:

$$\eta \frac{d^2U_2}{dY^2} + \frac{dU_2}{dY} \frac{\beta \sin \theta}{h_2} \frac{y}{dy} \frac{dU_1}{dy} + 2n \eta = 0;$$ \hspace{1cm} (16)

$$\eta \frac{d^2U_1}{dY^2} + \frac{dU_1}{dY} \frac{\beta \sin \theta}{h_2} \frac{y}{dy} \frac{dU_1}{dy} + 2 = 0.$$ \hspace{1cm} (17)

where,

\[ U_1 = \frac{u_1}{u_0}, \]
\[ U_2 = \frac{u_2}{u_0}, \]
\[ Y = \frac{y}{h_2}, \]
\[ \varphi = \frac{n h_2}{k_2}, \]
\[ u_0 = \frac{\rho g h_2^2 \sin \theta}{2 \mu}, \]
\[ \eta = \frac{\mu}{\beta \sin \theta}. \]

For the upper plant layer 4 and water layer 3, the coordinates of the top surface and the interface between the upper plant layer 4 and water layer 3 are \( z=0 \) and \( z=h_4 \). According to the equation (13), the control equations in the lower plant layer 4 and water layer 3 can be written as:

$$n\mu \frac{d^2u_4}{dz^2} + \frac{dn u_4}{dz} \frac{n \beta \sin \theta}{h_4} \frac{z}{dz} \frac{du_4}{dz} \hspace{1cm} (18)
- \frac{\mu n^2}{k_2} u_4 + n \rho g \sin \theta = 0,$$

$$\mu \frac{d^2u_3}{dz^2} + \frac{d(n \beta \sin \theta)}{h_4} \frac{z}{dz} \frac{du_3}{dz} + \rho g \sin \theta = 0.$$ \hspace{1cm} (19)

The dimensionless forms of the equations (18) and (19) are:

$$\zeta + Z \frac{d^2U_4}{dZ^2} + \frac{dU_4}{dZ} \frac{\beta \sin \theta}{h_4} \frac{z}{dz} \frac{dU_4}{dz} + 2n \zeta = 0.$$ \hspace{1cm} (20)
\[
\frac{d^2U_3}{dZ^2} + \frac{d}{dZ} \left( Z \frac{dU_3}{dZ} \right) + 2\zeta = 0 , \tag{21}
\]

where

\[
\zeta = \frac{\mu}{\beta \sqrt{\sin \theta}} h \frac{h}{h_m} - 1 \rangle = \eta \left( \frac{h}{h_m} - 1 \right) ,
\]

\[
U_1 = \frac{u_1}{u_m} ,
\]

\[
U_2 = \frac{n_m u_2}{u_m} ,
\]

\[
Z = \frac{z}{h_m} ,
\]

\[
\psi = \frac{n_m h_2}{k_4} .
\]

**III. VELOCITY DISTRIBUTION**

**A. Boundary conditions**

Following boundary conditions are considered in this study [12].

1. At the impervious bed \( Y = 0 \),

\[
U_2 = 0 \text{ (non-slip in \( x \) direction).} \tag{22}
\]

2. The interface between the lower plant layer 2 and water layer 1 \( Y = H_2 = h_1 / h_m \),

\[
U_1 = U_2 , \tag{23}
\]

\[
dU_1 \over dY = dU_2 \over n_m dY . \tag{24}
\]

3. The interface where the velocity reaches to the maximum value \( Y = 1 , Z = h / h_m - 1 \),

\[
dU_1 \over dY = 0 , \tag{25}
\]

\[
U_1 = U_3 , \tag{26}
\]

\[
dU_3 \over dZ = 0 . \tag{27}
\]

4. The interface between the upper plant layer 4 and water layer 3 \( Z = H_3 = h_3 / h_m \),

\[
U_3 = U_4 , \tag{28}
\]

\[
dU_3 \over dZ - dU_4 \over n_m dZ . \tag{29}
\]

5. At the top surface \( Z = 0 \),

\[
U_4 = 0 , \tag{30a}
\]

or \( \over dU_3 \over dZ = 0 . \tag{30b} \]

**B. Analytical solution of velocity**

Based on the control equations (16), (17) and Boundary conditions (22) ~ (26), the analytical solutions of longitudinal velocity in the water layer 1 and the lower plant layer 2 are

\[
U_1 = 2\eta \left[(1 + \eta) \ln \left( \frac{\eta + Y}{H_2 - Y} \right) + H_2 - Y \right] + \frac{2n_2}{\psi} + D_1 B_3 + D_2 B_6 \tag{31}
\]

\[
U_2 = \frac{2n_2}{\psi} + D_1 I_4 (2 \sqrt{\eta \psi} \sqrt{\eta + Y}) + \frac{D_4 K_5 (2 \sqrt{\eta \psi} \sqrt{\eta + Y})}{\psi} \tag{32}
\]

Based on the control equations (20), (21) and Boundary conditions (26)~(31), the analytical solutions of longitudinal velocity in the water layer 3 and the upper plant layer 4 are

\[
U_3 = 2\zeta \left[(H - 1 + \zeta) \ln \left( \frac{\zeta + Z}{H_3} \right) + H_4 - Z \right] + \frac{2n_3}{\psi} + D_3 A_4 + D_4 A_b \tag{33}
\]

\[
U_4 = \frac{2n_4}{\psi} + D_5 I_4 (2 \sqrt{\psi} \sqrt{\zeta + Z}) + \frac{D_5 K_5 (2 \sqrt{\psi} \sqrt{\zeta + Z})}{\psi} \tag{34}
\]

where:

\[
D_1 = \frac{2n_2 H_1 \sqrt{\eta}}{\sqrt{\psi} \eta + H_2} \frac{B_5 - B_6}{B_5 A_4 + B_2 B_3} \tag{35}
\]

\[
D_2 = \frac{2n_3 H_1 \sqrt{\eta}}{\sqrt{\psi} \eta + H_2} \frac{B_5 - B_6}{B_5 A_4 + B_2 B_3} \tag{36}
\]

\[
B_1 = I_6 (2 \sqrt{\psi}) \tag{37}
\]

\[
B_2 = K_6 (2 \sqrt{\psi}) \tag{38}
\]
\[ B_1 = I_1(2\sqrt{\eta \phi \eta + H_1}) , \]
\[ B_4 = K_1(2\sqrt{\eta \phi \eta + H_1}) , \]
\[ B_5 = I_0(2\sqrt{\eta \phi \eta + H_1}) , \]
\[ B_6 = K_0(2\sqrt{\eta \phi \eta + H_1}) , \]
\[ A_1 = I_1(2\zeta \sqrt{\phi \phi}) , \]
\[ A_2 = K_0(2\zeta \sqrt{\phi \phi}) , \]
\[ A_3 = I_1(2\sqrt{\zeta \psi \zeta + H_1}) , \]
\[ A_4 = K_1(2\sqrt{\zeta \psi \zeta + H_1}) . \]

\[ D_1 = -\frac{2n_2}{\phi} \frac{B_4}{B_1B_4 + B_2B_3} , \quad (44) \]
\[ D_2 = \frac{2n_2}{\phi} \frac{-B_3}{B_1B_4 + B_2B_3} . \quad (45) \]

(3) For the homogeneous flow, where \( h_2 = 0, h_3 = 0, h_4 = 0 \), the velocity distribution can be simplified into:

\[ U_1 = 2p(1 + \eta) \ln(1 + \frac{Y}{\eta}) - 2\eta Y . \quad (46) \]

IV. SHEAR STRESS DISTRIBUTION AND ENERGY RELATIONS

A. Vertical distribution of shear stress

The tangential shear stress in the lower plant layer 2 can be written as:

\[ \tau_2 = n_2 \frac{du_2}{dy} + n_2 \sqrt{\rho \sin \theta} \frac{y \, du_2}{h_m} \frac{dy}{h_m} , \quad (47) \]

where \( \tau_m = \rho g h_m \sin \theta \), according to the results of velocity \( (32) \), the vertical distribution shear stresses in the layer 2 is:

\[ D_2 = \frac{2n_2H_1 \sqrt{\zeta \psi \zeta + H_1}}{2\sqrt{\psi \zeta + H_1}} A_1 - A_4 \frac{2n_1}{\psi} , \quad (37) \]

\[ D_3 = \frac{2n_1H_1 \sqrt{\psi \zeta + H_1}}{2\sqrt{\psi \zeta + H_1}} A_1 - A_4 \frac{2n_1}{\psi} , \quad (38) \]

where, \( I_0(\nu) \) and \( I_1(\nu) \) are the modified Bessel function of the first kind, \( K_0(\nu) \) and \( K_1(\nu) \) are the modified Bessel function of the second kind.

\[ I_0(\nu) = \sum_{m=0}^{\infty} \frac{\nu^{2m}}{2^{2m}(m!)^2} , \quad (39) \]

\[ I_1(\nu) = \sum_{m=0}^{\infty} \frac{\nu^{2m+1}}{2^{2m+1}m!(m+1)!} , \quad (40) \]

\[ K_0(\nu) = -\sum_{m=0}^{\infty} \frac{\nu^{2m}}{2^{2m}(m!)^2} (\ln \frac{\nu}{2} - \psi(m+1)) , \quad (41) \]

\[ K_1(\nu) = \frac{1}{\nu} + \sum_{m=0}^{\infty} \frac{\nu^{2m+1}}{2^{2m+1}m!(m+1)!} (\ln \frac{\nu}{2} - \psi(m+1) - \frac{1}{2(m+1)}) , \quad (42) \]

\[ \psi(\nu) = -0.5772156 + \sum_{m=1}^{\infty} \frac{1}{m} . \quad (43) \]

C. Special cases

(1) For the submerged plant flow, i.e. \( h_1 = 0, h_4 = 0 \), the velocity in the water layer 2 and the submerged plant layer 1 are the equation \( (31) \) and \( (32) \).

(2) For the emergent plant flow, i.e. \( h_1 = 0, h_3 = 0, h_4 = 0 \), the analytical result of velocity in the plant layer is the equation \( (32) \), and the coefficients are:

\[ T_1 = \frac{\tau_1}{\tau_m} = 1 - Y , \quad (49) \]

\[ T_3 = \frac{\tau_3}{\tau_m} = H - 1 - Z , \quad (50) \]

\[ T_4 = \frac{\tau_4}{\tau_m} = \frac{2\sqrt{\zeta \psi \zeta + Z}}{2\sqrt{\zeta \psi \zeta + Z}} \left( D_1 I_1(2\sqrt{\zeta \psi \zeta + Z}) - D_2 K_1 \right) . \quad (51) \]

B. Energy relations

The energy relations for layer 2 can be derived by multiplying \( u_2 \) to Eq. (14)
\[ u_2 \left\{ n_2 \mu \frac{d^2 u_2}{dy^2} + \frac{d}{dy} \left( n_2 \beta \sqrt{\sin \theta} \frac{y}{h_m} \frac{du_2}{dy} \right) - \frac{\mu n_2^2}{k_2} u_2 + n_2 \rho g \sin \theta \right\} = 0 \] 

which can be rewritten in terms of shear stress by applying Eq. (47) 

\[ u_2 \frac{d \tau_2}{dy} - \frac{\mu}{k_2} n_2^2 u_2^2 + \rho g n_2 u_2 \sin \theta = 0. \] 

Considering the relation 

\[ \frac{u_2 \frac{d \tau_2}{dy} - \frac{\mu}{k_2} n_2^2 u_2^2 + \rho g n_2 u_2 \sin \theta}{\rho n_2^2 u_2^2} = 0. \] 

Substituting it into Eq. (53) and rearranging the result gives 

\[ \rho g n_2 u_2 \sin \theta = \frac{\mu}{k_2} n_2^2 u_2^2 + \tau_2 - \frac{d \tau_2}{dy}. \] 

The dimensionless form of Eq. (55) is the rate of energy supply by the resultant of gravity and pressure gradient 

\[ w_{b2} = \rho g n_2 u_2 \sin \theta. \] 

The first two terms on the right-hand-side of Eq. (55) is the rate of energy dissipation 

\[ w_{s2} = \frac{\mu}{k_2} n_2^2 u_2^2 + \tau_2 - \frac{d \tau_2}{dy}. \] 

where the first term is the dissipation rate due to the interaction between the pore water and plant, and the second term is the dissipation rate due to the fluid shear deformation. The last term on the right-hand-side of Eq. (55) is the rate of energy transfer in the \( y \) direction 

\[ w_{t2} = \frac{d \tau_2}{dy}. \] 

Their dimensionless forms are: 

\[ W_{b2} = \frac{w_{b2}}{w_m} = U_2, \] 

\[ W_{s2} = \frac{w_{s2}}{w_m} = \frac{\rho U_2^2}{n_2 \eta + Y}, \] 

\[ W_{t2} = \frac{w_{t2}}{w_m} = U_2 ^2 - \frac{\rho U_2^2}{n_2 \eta + Y}. \] 

where \( w_m = \rho g u_m \sin \theta \). The dimensionless velocities and shear stresses in layer 2 are given by the equations (32) and (48). 

The energy relations for layer 1, 3, 4 can be derived in the same way. Their dimensionless forms are: 

\[ W_{b1} = \frac{w_{b1}}{w_m} = U_1, \] 

\[ W_{s1} = \frac{w_{s1}}{w_m} = \frac{2 \eta T_1^2}{\eta + Y}, \] 

\[ W_{b3} = \frac{w_{b3}}{w_m} = U_3, \] 

\[ W_{s3} = \frac{w_{s3}}{w_m} = \frac{2 \xi T_3^2}{\xi + Z}, \] 

\[ W_{b4} = \frac{w_{b4}}{w_m} = U_4, \] 

\[ W_{s4} = \frac{w_{s4}}{w_m} = \frac{\psi U_4^2}{2 n_4} + \frac{2 \xi T_4^2}{n_4 \xi + Z}, \] 

\[ W_{t4} = \frac{w_{t4}}{w_m} = U_4 ^2 - \frac{2 \xi T_4^2}{n_4 \xi + Z}. \] 

The dimensionless velocities and shear stresses in layer 1, 3, 4 are given by the equations (31, 33, 36, 49, 50, 51), respectively.

V. RESULTS AND DISCUSSION

A. Introduction of parameters and verification

The parameters used in the results are introduced as follows: \( \rho = 10^3 \) kg/m\(^3\), \( \mu = 10^{-3} \) Pa·s, \( g = 9.8 \) m/s\(^2\). The special permeability \( k \) is calculated by using the Kozeny-Carman formula, 

\[ k = \frac{n^3}{C \eta T_0 S_s^2 (1-n)^2}, \] 

where \( C \) is shape factor and that it is equal 3 for the space between two plates\(^{[13]}\), \( T_0 \) is the tortuosity of porous medium, whose value is 2 suggested by Carman \(^{[14]}\). \( S_s \) is the specific surface area defined as the ratio of total surface area to total volume. We adopt \( S_s = 500 \) m\(^{-1}\). With Eq. (31), the dimensionless parameter becomes
\[ \varphi = \frac{n_1 h^2}{k_2} = C_4 T_0^2 S_2^2 h_0^2 \left( \frac{1}{n_2} - 1 \right)^2, \]  
\[ \psi = \frac{n_1 h^2}{k_4} = C_4 T_0^2 S_2^2 h_0^2 \left( \frac{1}{n_4} - 1 \right)^2, \]  

which shows that for \( n = 1 \) (homogeneous water), the parameters \( \varphi, \psi \) equal 0.

We use three sets of experimental data \( (C_1, C_2, C_3) \) of Zhang Xin to test the rationality of velocity distribution formula (31). In the experiment, flume width was 0.3m \( (B=0.3m) \), flume length was 8m \( (L=8m) \), depths \( (h_0) \) in three conditions were 0.061m, 0.0496m and 0.053m, the discharges were 0.01229 \( m^3/s \), 0.00875 \( m^3/s \) and 0.00875 \( m^3/s \), the bottom slope were 0.0025, 0.0025 and 0.0020.

Flow velocity was measured by laser Doppler anemometer. The calculated results of turbulent flow coefficient \( \beta \) are varied from 9.5 to 13.1. Velocity distribution is shown in Figure 3.

Form the figure 3, we can see the experimental and calculated results are coincident, and shows that the distribution formula can be reflected the flow velocity variation.

The parameters, such as \( n, T_0, C, S, \beta \) and so on, are generally required to determine by the experiment or experience. The following is the analysis of velocity distribution variation with these parameters.

![Figure 3. Comparison of velocity distribution verification](image_url)

**B. The effects of parameters on velocity distribution**

To qualitatively illustrate the parameter effecting on the velocity distribution, let us look at the variations of velocity distribution with the following dimensionless parameters: the coefficient of turbulence \( \eta, \zeta \), the factor of plants \( \varphi, \psi \), and the relative thicknesses of layers \( (H_1, H_2, H_3, H_4) \).

(1) The effects of the factor of plants \( \varphi \) or \( n \)

Figure 4 shows the effects of the plant factor on the velocity distribution with \( H_2=H_1=0.5, \eta=0.0001. \) \( n \) is the factor of plants. The velocity increases with increasing of the porosity \( n_2 \). The velocity distribution is the same as homogeneous flow for \( n_2=1 \).

(2) The effects of the relative thicknesses of layer \( (h_2/h_0) \)

Figure 5 shows the effects of the relative thicknesses of layer on the velocity distribution with \( n_2=0.95, \eta=0.0001. \) The velocity increases with the increase of the relative thickness \( H_1 \) or the decrease of the relative thickness \( H_2 \). The velocity distribution is the same as homogeneous flow for \( H_2=0 \) and \( H_1=1 \). The velocity distribution is the same as emergent plant flow for \( H_2=1 \) and \( H_1=0 \).

(3) The effects of the coefficient of turbulence \( \eta \)

Figure 6 shows the effects of turbulence on the velocity distribution with \( n_2=0.95, H_2=H_1=0.5. \) The coefficient \( \eta \) or \( \beta \) denotes the factor of turbulence. The velocity distribution gets uniform and the dimensionless velocity increases with the increase of the turbulent coefficient \( \eta \).

![Figure 4. Velocity distributions in various parameter (n)](image_url)

![Figure 5. Velocity distributions in various parameter \((H_2=1-H_1)\)](image_url)

![Figure 6. Velocity distributions in various parameter \((\eta)\)](image_url)
The velocity increases with increasing of the porosity $n_2$ for $n_4 = 0.95$. With an increase in the porosity $n_2$, the point of the maximum velocity locate $y/h = 0.6245$, 0.50 and 0.3223, the maximum dimensionless velocities are $1.80 \times 10^{-4}$, $4.51 \times 10^{-4}$ and $1.21 \times 10^{-3}$, respectively.

Figure 7. Velocity distributions in various parameter ($n_2$, $n_4$)

C. The effects of parameters on shear stress distribution

Figure 8 shows the distribution of shear stress with $n_2 = 0.98$, $n_4 = 0.95$, $\eta = 0.0001$. We can see that:

1) For the homogeneous layer 1 and 3, the distribution of shear stress follows the linear law with the slope of -1. The shear stress is zero at the interface between the homogeneous layer 1 and 3 where the velocity reaches to the maximum value (Figure 2);

2) For the submerged plant layer 2, the shear stress is slightly affected by the submerged plants because its porosity $n_2 = 0.98$ is near 1.

3) For the floating plant layer 4, the shear stress is significantly affected by the floating plants because its porosity $n_3 = 0.95$ is more less. At the interface of plant layer and homogeneous water layer, the shear stress is not continuous. The continuous condition of shear stress at the interface of layer 1 and layer 2 is:

$$T_1 = \frac{T_{t_1}}{n_2}. \tag{74}$$

The continuous condition of shear stress at the interface of layer 3 and layer 4 is:

$$T_3 = \frac{T_{t_3}}{n_4}. \tag{75}$$

D. The effects of parameters on energy relations

As a part of the whole project, Huai et al$^{[15,16]}$ recently discussed the mechanism of energy loss and transition in a flow with submerged vegetation. In the following, the effect of the floating and submerged aquatic plants is discussed. To limit the length of this discussion, only one case ($n_2 = 0.98$, $n_4 = 0.95$, $\eta = 0.0001$) is discussed. According to equations (59) - (70), the distributions of rates of energy supply and dissipation are plotted in Figure 9, where the difference between them is the energy transfer. We can see that: (1) In the homogeneous water layer 1 and 3, the energy supply rate is much greater than the energy dissipation rate, which means the surplus of energy supply must be transferred to the plant layer. (2) In the floating plant layer 4, the energy supply rates are much smaller than that of energy dissipation, which means to maintain their flows, the plant layer must gain energy from the homogeneous water layer. (3) In the submerged plant layer 2, the sum of the energy supply is greater than that of energy dissipation, which means that the surplus energy in the submerged plant layer 4 will transfer to the floating plant layer 4. (4) At the porous interfaces, there exists an abrupt change of energy dissipation. This is not surprising because, physically, the solid skeleton in the porous media suddenly applies a large resistance to the flow. (5) The total energy supply is equal to the total energy dissipation; the energy transfer term just redistributes the energy between the local supply and dissipation over a cross-section. Similar results can be found for other cases although they are not plotted here.

Figure 9 Energy relations in wetlands flows with porosity

VI. CONCLUSION

By applying Boit’s poroelastic theory to turbulent flow in wetlands with submerged and floating aquatic plants, the following conclusions are drawn:

1) The momentum equation in the flow direction for turbulent flows is given. In the equation, the driving force is the effective gravity component in the flow direction, while the resistance includes the viscous shear stress and the Reynolds shear stress.
2) The dimensionless vertical distributions of velocity for different layers are deduced. The analytical results can be simplified for various cases. The velocity reaches its maximum value in the homogeneous water layer, decays rapidly in the plant layers, and decreases with decreasing plant porosity. The dimensionless velocities are continuous at the porous interfaces in terms of volume average.

3) The shear stress are analyzed and the dimensionless value for the homogeneous water layer distributes linearly with a slope of -1; the plant layer with larger porosity has little effect on the shear stress; the plant layer with less porosity is shifts and reduces the linear shear stress distribution in the water layer. The shear stress is zero at the interface between the homogeneous layer 1 and 3 where the velocity reaches to its maximum value. At the interface of plant layer and homogeneous water layer, the shear stress is not continuous. In other words, there exists a slipping shear stress between the homogeneous water and the pore water in the porous media.

4) The energy relations in a wetland flow are described. The energy supply is balanced by the energy dissipation and the energy transfer. The expressions of dimensionless energy supply, dissipation and transfer are deduced. The homogeneous water layer supplies most of the energy; the plant layer 4 with less porosity dissipate most of the energy by the interaction between the pore water and the solid skeleton; The total energy supply is equal to the total energy dissipation over a cross-section, and the energy transfer term just redistributes the cross-sectional energy according to the local supply and dissipation.

5) The results of this study are expected to help water resources, environmental and ecological engineers design and manage surface water flows in wetlands.

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